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Evidence for time-reversal violation?

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Abstract. The experiment CPLEAR at CERN has measured an asymmetry between the transition rates of the antikaon to kaon process, and its reverse, kaon to antikaon. The present note shows, based on a criterion by Wolfenstein, that this observation constitutes direct evidence for time-reversal violation. Experiments at FNAL (KTEV) and CERN (NA48) have measured a T-odd asymmetry in the K_{L} decay. Based on an analysis by Sehgal and van Leusen, it is demonstrated that this asymmetry is well described by a (hypothetical) kaon time evolution, which is time-reversal invariant.

Two experiments (CPLEAR at CERN [1] and E799 at FNAL [2]) have measured asymmetries in kaon decay, and have claimed to have observed *direct evidence for time-reversal symmetry violation* (TRV), for the first time. Several authors [3–7] have examined (and sometimes questioned) the validity of these conclusions. Thanks to a formal criterion for "direct evidence for TRV" due to Wolfenstein [7], it has become possible to decide on these questions in a well-defined way.

This note demonstrates three issues.
(i) Using the standard treatment of the neutral kaon system, the experimental asymmetry $A_{\mathrm{CPLEAR}} \neq 0$, found at CERN, satisfies the Wolfenstein criterion, and thus constitutes direct evidence for time-reversal violation.
(ii) As it has been considered "doubtful [3] that any decay experiment by itself can provide direct evidence for TRV", we discuss the consistency of the Wolfenstein criterion with the standard formalism for the neutral kaon time evolution. We find an inequality and a linear condition on the parameters, both harmless for the advent of TRV.
(iii) The T-odd asymmetries observed in the KTEV and NA48 experiments are well described by a (hypothetical) *time-reversal invariant* time evolution of the neutral kaons, based on the analysis in [4]. My conclusions are at variance with some ones of [3, 4, 6].

The standard treatment of the time evolution of neutral kaons has been given in many places, e.g. in [8]. We mention a few points to fix our definitions. The time development of a neutral kaon is described by $\psi(t) = \exp(-\mathrm{i}\Lambda t)\psi(0)$. ψ has two components, the amplitudes for the particle to be a kaon or to be an antikaon. The elements Λ_{ij} $(= \langle i|\Lambda|j\rangle)$ of the 2×2 matrix Λ are complex constants, the parameters of the phenomenology. In the decomposition, $\Lambda = M - 1/2\mathrm{i}\Gamma, M = M^{+}, \Gamma = \Gamma^{+}$, Γ has to be a non-negative matrix [9]. Time-reversal invariance is defined as the property $\mathcal{T}\mathcal{H}_{\mathrm{w}}\mathcal{T}^{-1} = \mathcal{H}_{\mathrm{w}}$ of the weak interaction Hamiltonian $\mathcal{H}_{\mathrm{w}}$ under the $\mathcal{T}$ transformation. This implies $\Gamma_{12}^{*}/\Gamma_{12} = M_{12}^{*}/M_{12}$ as the corresponding property of Λ [8], from which we deduce the equivalent relation $|\Lambda_{12}|^2 = |\Lambda_{21}|^2$. We shall use [10]

$$A_{\mathrm{Kabir}} \equiv (|\Lambda_{12}|^2 - |\Lambda_{21}|^2)/(|\Lambda_{12}|^2 + |\Lambda_{21}|^2) \qquad (1)$$

as the theoretical measure for TRV.

The CPLEAR experiment at CERN has measured the asymmetry of the probabilities for an antikaon to become a kaon within time t, B, compared with that for a kaon to become an antikaon, F,

$$A_{\mathrm{CPLEAR}} \equiv (B - F)/(B + F)\,, \qquad (2)$$

where

$$B = |\langle K^0|\exp(-\mathrm{i}\Lambda t)|\bar{K}^0\rangle|^2$$

and

$$F = |\langle \bar{K}^0|\exp(-\mathrm{i}\Lambda t)|K^0\rangle|^2\,.$$

The calculation (discussed below) yields

$$A_{\mathrm{CPLEAR}} \equiv A_{\mathrm{Kabir}}. \qquad (3)$$

Adopting these definitions (and thereby avoiding the celebrated, but convention-dependent parameter "ϵ") we are able to use relations only, which are invariant with respect to the arbitrary value of an unobservable relative phase between the kaon and antikaon states.

The experiment has obtained $A_{\mathrm{CPLEAR}} \neq 0$. The Wolfenstein criterion serves to decide whether the measured finite asymmetry A_{CPLEAR} is indeed a *direct* consequence of TRV, as suggested by (3), or whether it merely reflects the big difference $\Delta\Gamma = \gamma_{\mathrm{S}} - \gamma_{\mathrm{L}}$ of the decay rates of the eigenstates of Λ, $|K_{\mathrm{S}}\rangle$ and $|K_{\mathrm{L}}\rangle$, and thus might be a doubtful manifestation of TRV. The criterion [7] defines that *"direct evidence for TRV" would exist, if, in the limit of a vanishing decay-rate difference $\Delta\Gamma \to 0$, (3) would* *remain valid.* To analyse this question, we consider a simple deduction of (3): Following [11], Λ is represented as a superposition of Pauli matrices

$$\Lambda = \Lambda_\mu \sigma^\mu , \tag{4}$$

with σ^0 = unit matrix, σ^k = Pauli matrices. (Summation over multiple indices: Greek: 0 to 3, Roman: 1 to 3). We apply the generalized Euler formula

$$\exp(-\mathrm{i}\Lambda_\mu \sigma^\mu t) = \exp(-\mathrm{i}\Lambda_0 t) \times (\sigma^0 \cos(\Omega t) - \mathrm{i}\Lambda_m \sigma^m (1/\Omega) \sin(\Omega t)) \tag{5}$$

(where $\Omega = (\Lambda_m \Lambda_m)^{1/2} = 1/2(\lambda_\mathrm{L} - \lambda_\mathrm{S}) = 1/2(\Delta m + 1/2\mathrm{i}\Delta\Gamma)$, and where λ_L, λ_S are the eigenvalues of Λ; $\Delta m = m_\mathrm{L} - m_\mathrm{S}$ is the mass difference of the eigenstates) to A_CPLEAR and obtain

$$\begin{aligned} B &= G(t, \Delta m, \Delta\Gamma)|\Lambda_{12}|^2 , \\ F &= G(t, \Delta m, \Delta\Gamma)|\Lambda_{21}|^2 , \\ G(t, \Delta m, \Delta\Gamma) &= |(1/\Omega)\sin(\Omega t)|^2 \\ &= 2(\cosh(1/2\Delta\Gamma t) \\ &\quad - \cos(\Delta m t))/((\Delta m)^2 \\ &\quad + (1/2\Delta\Gamma)^2) , \end{aligned} \tag{6}$$

and finally, $A_\mathrm{CPLEAR} = (|\Lambda_{12}|^2 - |\Lambda_{21}|^2)/(|\Lambda_{12}|^2 + |\Lambda_{21}|^2) \equiv A_\mathrm{Kabir}$. We note that $G(t, \Delta m, \Delta\Gamma)$ is a well behaved function for all $\Delta\Gamma < \infty$. In the expression for A_CPLEAR, $\Delta\Gamma$ drops out. Equation (3) *is thus a mathematical identity for any* $\Delta\Gamma$, and $A_\mathrm{CPLEAR} \neq 0$ is a direct consequence of $A_\mathrm{Kabir} \neq 0$, i.e. of TRV.

In order to study the implications of the Wolfenstein criterion on the theoretical values for TRV, we calculate A_Kabir in terms of the reals M_m and Γ_m, defined by $\Lambda = M_\mu \sigma^\mu - 1/2\mathrm{i}\Gamma_\mu \sigma^\mu$, and we obtain

$$\begin{aligned} A_\mathrm{Kabir} &= 2\mathrm{Im}(\Lambda_1^* \Lambda_2)/(|\Lambda_1|^2 + |\Lambda_2|^2) \\ &= (\Gamma_1 M_2 - \Gamma_2 M_1)/(M_1^2 + M_2^2 + 1/4(\Gamma_1^2 + \Gamma_2^2)). \end{aligned} \tag{7}$$

As a side remark, we note that the non-negativity of Γ (which serves to avoid unphysical values for probabilities [9]) demands for $\Gamma_0 \geq 0$, and for $\Gamma_0\Gamma_0 \geq \Gamma_m\Gamma_m$, with the consequence that TRV ($A_\mathrm{Kabir} \neq 0$) requires $\Gamma_0 > 0$, i.e. that the neutral kaons have to decay.

The restriction $\Delta\Gamma = 0 (= 4\mathrm{Im}(\Omega))$ requires $\Lambda_m\Lambda_m$ = real, non-negative. Inserting M_m and Γ_m, we find that

$$\Delta\Gamma = 0 \tag{8}$$

if and only if

$$M_m M_m \geq 1/4\Gamma_m\Gamma_m \tag{9}$$

and

$$M_m \Gamma_m = 0. \tag{10}$$

For a given value of A_Kabir determined by (7), we are always able to formally satisfy (9) and (10).

It has also been speculated [7] that the constancy with time of A_CPLEAR is related to $\Delta\Gamma$. However, our derivation of (3) shows that this constancy is of *purely mathematical* origin, and reflects that the neutral kaon is represented in a space of *two* dimensions. From (5), we note as a corollary that

$$(\exp(-\mathrm{i}\Lambda t))_{i\neq j} = (-\mathrm{i}\Lambda t)_{i\neq j} \exp(-\mathrm{i}\Lambda_0 t)\sin(\Omega t)/(\Omega t) , \tag{11}$$

i.e. *the off-diagonal elements of the exponential of any* 2×2 *matrix are proportional to the off-diagonal elements of the exponent matrix, with equal factors.*

This makes A_CPLEAR constant in time. It also explains the basic difference to the appearance of a possible TRV in the neutrino oscillations (ν_e, ν_μ, ν_τ), as this system's representation space is of higher dimension.

The large T-odd asymmetry $\gamma_3 \sin\phi\cos\phi$ in the distribution of the angle ϕ between the lepton plane and the pion plane in the decay $K_\mathrm{L} \to \pi^+\pi^-e^+e^-$, as observed in the KTEV experiment, and recently confirmed by the NA48 collaboration at CERN [12], is shown to originate from a large component of the Stokes vector (called S_1 in [4]), and *not to vanish* in the hypothetical case $\Gamma_{12} \to 0$. In contrast, we note that $A_\mathrm{Kabir} = 2\mathrm{Im}(M_{12}^*\Gamma_{12})/(|\Lambda_{12}|^2 + |\Lambda_{21}|^2)$ *vanishes* if $\Gamma_{12} \to 0$. The authors of [4] have thus demonstrated that the T-odd asymmetry, based on S_1, *survives* if the TRV is *switched off*, and thus cannot be a manifestation of TRV in the time evolution of the neutral kaon.

To summarize: we have shown, confirming [1], that $A_\mathrm{CPLEAR} \neq 0$ follows from T-violation, while $\gamma_3 \neq 0$ does not.

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